Deriving quantum mechanics from statistical assumptions

U. Klein

University of Linz, Institute for Theoretical Physics, A-4040 Linz, Austria

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This talk is about the interpretation of Quantum Mechanics (QM)

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- What is the meaning of the qm formalism?
- It is hard to understand in comparison to classical physics!
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What do we mean by classical physics?

What exactly do we mean (or should we mean) by classical physics?

Common answer: classical mechanics - a deterministic theory describing the movement of point particles.

Let us characterize the common answer by the term "particle picture".
What exactly do we mean (or should we mean) by *classical physics*?

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Let us characterize the common answer by the term "particle picture".
There are two different (but possibly related) manifestations of the particle picture:

- The formal transition from classical physics to quantum mechanics, i.e. the *quantization procedure*, and
- the *interpretation* of the experimental data.
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- The formal transition from classical physics to quantum mechanics, i.e. the \textit{quantization procedure}, and
- the \textit{interpretation} of the experimental data.
In the particle picture the \textit{Hamilton function} $H(p, q)$ of classical \textit{particle physics} has to be used as a starting point.

The "canonical" quantization procedure:

Replace the classical formula $H(p, q) = E$ by $H(\hat{p}, \hat{q})\psi = \hat{E}\psi$, where $\hat{q} = q$, and

$$\hat{p} = \frac{\hbar}{i} \frac{d}{dx}, \quad \hat{E} = -\frac{\hbar}{i} \frac{d}{dt}.$$
The particle picture represents the prevailing opinion. Despite of that it may be allowed to ask the following two questions:
1 Is it the only possible choice?

Answer: No, QM could as well be compared to a classical statistical theory.
A statistical theory (as defined here) is not deterministic with regard to single events.
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Second question

2 Is it clear and simple?

Answer: No, certainly not

- The above quantization procedure is *incomprehensible*.
- Experimental data (interpreted in the particle picture) are *mysterious*.
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- Experimental data (interpreted in the particle picture) are *mysterious*.
Where do all the mysteries come from?

Why does the particle picture lead to mysterious results?

Answer: The qm formalism is *unable to make predictions about individual events*. Only probabilities are provided. Thus, there is a clash between the available theory and our desire for a *complete* theory (complete means here simply: deterministic with regard to single events).
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How to escape the clash?

There are two strategies to escape the clash between wish and reality:

- Complement QM by additional intellectual constructs, preferably of a highly abstract (in a philosophical and/or mathematical sense) character.

- Give up the particle picture – i.e. consider QM as a purely statistical theory (basically Einsteins view of QM, see Ballentines article in Reviews of Modern Physics).
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In the rest of this talk I will try to make a point in favor of this Statistical Interpretation of QM.

The incomprehensibility of the quantization procedure and the mysterious nature of the experimental data are different aspects of one and the same thing.

I assume that the particle picture is the "wrong" starting point for quantization, and that a statistical theory should be used instead.

A successful "simple" quantization (derivation of QM) from statistical postulates would also present a strong argument in favor of the Statistical Interpretation of QM.
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The theory, general features

- Classical statistical physics contains both deterministic and indeterministic elements. The idea is to remove the deterministic elements.
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A possible rule to obtain a fundamental condition for such "generalized" statistical theories: start from a particle theory and then, 

\textit{replace in the particle theory all observables by ensemble averages.}

This leads to our "first assumption" (see next frame)

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The first assumption

The "true dynamical law" for a particle in a force field

\[ F(x) = -\frac{dV(x)}{dx}, \]

is not

\[ \frac{d}{dt}x(t) = \frac{p(t)}{m}, \quad \frac{d}{dt}p(t) = F(x(t)), \]

but

\[ \frac{d}{dt}\overline{x} = \frac{\overline{p}}{m}, \quad \frac{d}{dt}\overline{p} = \overline{F(x)}, \]

where \(\overline{x}, \overline{p}\) and \(\overline{F}\) are ensemble averages.
The quantities $x, p$ are not observables but random variables.

The ensemble averages are given by

$$
\bar{x} = \int_{-\infty}^{\infty} dx \: \rho(x, t) \: x, \quad \bar{p} = \int_{-\infty}^{\infty} dp \: w(p, t) \: p, \ldots \text{etc,}
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with the probabilities $\rho(x, t)$ and $w(p, t)$ playing the role of the new "observables".

No law is yet known for $\rho(x, t)$ and $w(p, t)$. Many different laws are possible $\implies$ the first assumption may be referred to as "statistical condition(s)".
Some comments on the first assumption

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The second assumption

A conservation law for the probability density $\rho(x, t)$,

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial}{\partial x} \frac{\rho(x, t)}{m} p(x, t) = 0,$$

exists, where the momentum field $p(x, t)$ can be written as a gradient of a scalar function $S(x, t)$

$$p(x, t) = \frac{\partial S(x, t)}{\partial x}.$$

The momentum field $p(x, t)$ and the random variable $\rho$ are (generally) different!
We need a third assumption

The above two assumptions lead to an infinite number of statistical theories. [Differential equations for $\rho(x, t)$ and $S(x, t)$].

These theories may be labelled by an (almost) arbitrary function $L$. Special cases are QM (for $L = L^q$) and a classical statistical theory, which is the classical limit of QM (for $L = 0$).

Why is $L = L^q$ realized and not any other choice? Which principle has been used by nature?
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The third assumption

We hope that something like the principle of maximal entropy (in statistical thermodynamics) might work here, and postulate:

- The principle used by nature is the principle of maximal disorder, as realized by the principle of minimal Fisher information.
These assumptions imply Schrödinger’s equation

The above three assumptions imply

\[-\frac{\hbar}{i} \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V \psi,\]

where

\[\psi = \sqrt{\rho} e^{i \frac{S}{\hbar}}.\]

Two open questions?

- \(w(p, t)\)?
- How to obtain expectation values of \(p\)–dependent quantities?
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We constructed a "configuration space" probabilistic theory, i.e.:
The sample space is $\mathcal{R}$, the set of possible outcomes of $x$.

On the other hand:

- The momentum $p$ of the present theory is a random variable.
- But this random variable $p$ is not defined as a function of $x$ - like a random variable of "classical" probability theory.
- This fact presents the essential nonclassical feature of the present approach to QM - it means that a deterministic element must be removed from standard probability theory in order to obtain QM.
- To obtain $w(p, t)$ and to answer the above questions one more condition must be implemented.
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Assumption no. 4 is *energy conservation in the mean*:

\[
\frac{d}{dt} \left[ \int_{-\infty}^{\infty} dp w(p, t) \frac{p^2}{2m} + \int_{-\infty}^{\infty} dx \rho(x, t) V(x) \right] = 0.
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- Assumptions 1-4 imply:
  - As far as the calculation of expectation values of \( p^n \) for \( n = 0, 1, 2 \) is concerned, the probability density is given by
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    w(p, t) = \frac{1}{\hbar} |\phi(p, t)|^2,
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Summary

- We found a set of assumptions which imply QM. All assumptions may be interpreted in physical terms.
- This *comprehensible* quantization procedure does not start from a classical particle Hamiltonian but from a (abstract) statistical theory.
- It gives an explanation for the success of the canonical quantization procedure - for rules like $p \rightarrow \frac{\hbar}{i} \frac{d}{dx}$.
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The central new assumption (no.1) was a set of two dynamical equations with the structure of classical particle mechanics - but with ensemble averages replacing observables.

QM is a configuration space probabilistic theory with a new element of indeterminism (momentum no longer standard random variable) added.

All this gives very strong support to the Statistical Interpretation of QM.
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