Deriving quantum mechanics from statistical assumptions

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Joint Annual Meeting of ÖPG/SPS/ÖGAA - Innsbruck 2009

The interpretation of quantum mechanics

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- What is the meaning of the qm formalism ?
- It is hard to understand in comparison to *classical physics* !

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- Common answer: *classical mechanics* - a deterministic theory describing the movement of point particles.

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Two manifestations of the particle picture.

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- The formal transition from classical physics to quantum mechanics, i.e. the *quantization procedure*, and
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The quantization procedure in the particle picture

In the particle picture the *Hamilton function $H(p, q)$ of classical particle physics* has to be used as a starting point.

The "canonical" quantization procedure:

Replace the classical formula $H(p, q) = E$ by $H(\hat{p}, \hat{q})\psi = \hat{E}\psi$, where $\hat{q} = q$, and

$$\hat{p} = \frac{\hbar}{\imath}\frac{\mathrm{d}}{\mathrm{d}x}, \quad \hat{E} = -\frac{\hbar}{\imath}\frac{\mathrm{d}}{\mathrm{d}t}.$$

Calling the particle picture into question

The particle picture represents the prevailing opinion. Despite of that it may be allowed to ask the following two questions:

First question

1. Is it the only possible choice ?

   Answer: No, QM could as well be compared to a classical *statistical* theory.
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Answer: No, certainly not

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Where do all the mysteries come from ?

- Why does the particle picture lead to mysterious results ?

  Answer: The qm formalism is *unable to make predictions about individual events*. Only probabilites are provided.

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How to escape the clash ?

There are two strategies to escape the clash between wish and reality:

- Complement QM by additional intellectual constructs, preferably of a highly abstract (in a philosophical and/or mathematical sense) character.
- Give up the particle picture – i.e. consider QM as a purely statistical theory (basically Einsteins view of QM, see Ballentines article in Reviews of Modern Physics).

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A calculation supporting an interpretation

- In the rest of this talk I will try to make a point in favor of this Statistical Interpretation of QM.
- The incomprehensibility of the quantization procedure and the mysterious nature of the experimental data are different aspects of *one and the same thing*.
- I assume that the particle picture is the "wrong" starting point for quantization, and that a statistical theory should be used instead.
- A successful "simple" *quantization* (derivation of QM) from statistical postulates would also present a strong argument in favor of the Statistical *Interpretation* of QM.

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- in a preprint to be uploaded to the Arxiv e-print repository in the near future
- in parts of a draft at Arxiv, which is, however, incomplete and inconclusive,
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How to obtain the fundamental dynamical laws ?

- A possible rule to obtain a fundamental condition for such "generalized" statistical theories:
  start from a particle theory and then,

  replace in the particle theory all observables by ensemble averages.

  This leads to our "first assumption" (see next frame)
- We study a single classical particle in a single spatial dimension x .

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The first assumption

The *"true dynamical law"* for a particle in a force field

$$F(x) = -\frac{\mathrm{d}V(x)}{\mathrm{d}x},$$

is not

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = \frac{p(t)}{m}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}p(t) = F(x(t)),$$

but

$$\frac{\mathrm{d}}{\mathrm{d}t}\overline{x} = \frac{\overline{p}}{m} \qquad \frac{\mathrm{d}}{\mathrm{d}t}\overline{p} = \overline{F(x)},$$

where $\overline{x}$, $\overline{p}$ and $\overline{F}$ are *ensemble averages*.

Some comments on the first assumption

- The quantities x, p are not observables but random variables.
- The ensemble averages are given by

$$\overline{x} = \int_{-\infty}^{\infty} \mathrm{d}x\, \rho(x,t)\, x, \quad \overline{p} = \int_{-\infty}^{\infty} \mathrm{d}p\, w(p,t)\, p, \text{...etc},$$

- with the probabilities $\rho(x,t)$ and $w(p,t)$ playing the role of the new "observables".
- No law is yet known for $\rho(x,t)$ and $w(p,t)$. *Many different laws* are possible $\Longrightarrow$ the first assumption may be referred to as "statistical condition(s)".

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The second assumption

A conservation law for the probability density $\rho(x,t)$,

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial}{\partial x} \frac{\rho(x,t)}{m} p(x,t) = 0,$$

exists, where the momentum field $p(x,\ t)$ can be written as a gradient of a scalar function $S(x,\ t)$

$$p(x,\ t) = \frac{\partial S(x,\ t)}{\partial x}.$$

The momentum field $p(x,\ t)$ and the random variable p are (generally) different !

We need a third assumption

- The above two assumptions lead to an infinite number of statistical theories. [Differential equations for $\rho(x,t)$ and $S(x, t)$].
- These theories may be labelled by an (almost) arbitrary function L. Special cases are QM (for $L = L^q$) and a classical statistical theory, which is the classical limit of QM (for $L = 0$).
- Why is $L = L^q$ realized and not any other choice ? Which principle has been used by nature ?

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The third assumption

We hope that something like the principle of maximal entropy (in statistical thermodynamics) might work here, and postulate:

- The principle used by nature is the principle of maximal disorder, as realized by the principle of *minimal Fisher information*

These assumptions imply Schrödinger's equation

The above three assumptions imply

$$-\frac{\hbar}{\imath}\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi,$$

where

$$\psi = \sqrt{\rho}\mathrm{e}^{\imath\frac{S}{s}}.$$

- Two open questions ?
- $w(p,t)$?
- How to obtain expectation values of $p-$dependend quantities ?

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The momentum random variable

We constructed a "configuration space" probabilistic theory, i.e.: The sample space is $\mathcal{R}$, the set of possible outcomes of x.

On the other hand:

- The momentum p of the present theory is a random variable.
- But this random variable p is not defined as a function of x - like a random variable of "classical" probability theory.
- This fact presents the essential nonclassical feature of the present approach to QM - it means that a

  deterministic element must be removed from standard probability theory in order to obtain QM.
- To obtain $w(p,t)$ and to answer the above questions one more conditon must be implemented.

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Energy conservation in the mean

Assumption no.4 is *energy conservation in the mean*:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\int_{-\infty}^{\infty}\mathrm{d}p\,w(p,t)\frac{p^2}{2m}+\int_{-\infty}^{\infty}\mathrm{d}x\,\rho(x,t)V(x)\right]=0.$$

- Assumptions 1-4 imply:
  As far as the calculation of expectation values of p^n for $n=0,1,2$ is concerned, the probability density is given by

$$w(p,t)=\frac{1}{\hbar}|\phi(p,t)|^2,$$

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- No values of n different from $n=0,1,2$ seem to exist in realistic situations.

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Summary

- We found a set of assumptions which imply QM. All assumptions may be interpreted in physical terms.
- This *comprehensible* quantization procedure does not start from a classical particle Hamiltonian but from a (abstract) statistical theory.
- It gives an explanation for the success of the canonical quantization procedure - for rules like $p \longrightarrow \frac{\hbar}{\imath}\frac{\mathrm{d}}{\mathrm{d}x}$

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- The central new assumption (no.1) was a set of two dynamical equations with the structure of classical particle mechanics - but with ensemble averages replacing observables.
- QM is a configuration space probabilistic theory with a new element of indeterminism (momentum no longer standard random variable) added.
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